Problem 4.1: Let $X_t = A \cos(Wt + Y)$ where $A$, $W$, and $Y$ are independent random variables. $A$ has mean 2 and variance 4, $Y$ is uniform on $[-\pi, \pi]$ and $W$ is uniform on $[0, 5]$. Find the mean function $\mu_X(t)$ and autocorrelation function $R_X(s, t)$. Is $X$ wide sense stationary?

Problem 4.2: Let $Y$ and $Z$ be independent random processes with $R_Y(s, t) = 2 \exp(-|s - t|) \cos(2\pi f(s - t))$ and $R_Z(s, t) = 9 + \exp(-3|s - t|^4)$. Find the autocorrelation function $R_X(s, t)$ where $X_t = Y_t Z_t$.

Problem 4.3: Suppose that $X_1$ and $X_2$ are random variables such that $E[X_1] = E[X_2] = E[X_1 X_2] = 0$ and $\text{Var}(X_1) = \text{Var}(X_2) = \sigma^2$. Define $Y_t = X_1 \sin(t) + X_2 \cos(t)$.

(a) Is the random process $Y$ necessarily wide-sense stationary?

(b) Give an example of random variables $X_1$ and $X_2$ satisfying the given conditions such that $Y$ is not (strict sense) stationary.

(c) Give an example of random variables $X_1$ and $X_2$ satisfying the given conditions such that $Y$ is stationary.

Problem 4.4: Define a random process $X$ by $X_t = A + Bt + t^2$ where $A$ and $B$ are independent, $N(0, 1)$ random variables.

(a) Find $\hat{E}[X_5|X_1]$, the linear minimum mean square error (LMMSE) estimator of $X_5$ given $X_1$, and compute the mean square error.

(b) Find the MMSE (possibly nonlinear) estimator of $X_5$ given $X_1$, and compute the mean square error. (Hint: Can be done by inspection).

Problem 4.5: Define the random process $X$ by $X_t = 2A + Bt$ where $A$ and $B$ are independent random variables with $P[A = 1] = P[A = -1] = P[B = 1] = P[B = -1] = 0.5$.

(a) Sketch the possible sample functions.

(b) Find $P[X_t \geq 0] \forall t$.

(c) Find $P[X_t \geq t] \forall t$.

Problem 4.6: Stark & Woods 6.43 (Page 399)

Problem 4.7: Stark & Woods 7.3 (Page 470)

Problem 4.8: Stark & Woods 7.5 (Page 471)

Problem 4.9: Stark & Woods 7.9 (Page 473)

Problem 4.10: Stark & Woods 7.17 (Page 476)

Problem 4.11: Stark & Woods 7.20 (Page 477)