Lab Assignment 4: Transform analysis of LTI systems

L4.1: Suppose the system function of a causal system is given by

\[ H(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}. \]

Assume \( a = r e^{j\theta} \) (with \( r < 1 \)), a) find the magnitude, phase, and group delay of \( H(z) \); b) Use Matlab to plot the log magnitude, phase, and group delay of the system defined in problem 5.8 for \( a = 0.8 \) and \( a = -0.8 \).

L4.2: A lowpass FIR filter is given as:

\[ h[0] = \frac{(1+\sqrt{3})}{4\sqrt{2}}, h[1] = \frac{(3+\sqrt{3})}{4\sqrt{2}}, h[2] = \frac{(3-\sqrt{3})}{4\sqrt{2}}, h[3] = \frac{(1-\sqrt{3})}{4\sqrt{2}}, \text{ and } h(n) = 0 \]

for other \( n \). Now define a highpass FIR filter \( g[n] \) based on \( h[n] \) as \( g[n] = (-1)^n h[n] \). We know that the DC gain of \( h[n] \) is \( H(e^{j\omega})|_{\omega=0} = \sum_{n=-\infty}^{\infty} h[n] = \sqrt{2} \) and \( G(e^{j\omega}) = H(e^{j(\omega+\pi)}) \). It also can be shown that \( |H(e^{j\omega})|^2 + |G(e^{j\omega})|^2 = 2 \). In other words, \( |H(e^{j\omega})|^2 + |H(e^{j(\omega+\pi)})|^2 = 2 \). This is one of the conditions that \( h[n] \) has to satisfy in order to be called an orthogonal wavelet filter.

(a) Use “freqz” in Matlab to plot the log magnitude and phase of \( h[n] \) and \( g[n] \).

(b) Plot \( |H(e^{j\omega})|^2 + |H(e^{j(\omega+\pi)})|^2 \) and check if it is indeed a constant that equals to 2.